scisearch INTERNATIONAL PEER-REVIEWED JOURNALS

RESEARCH PAPER

Evaluation of GNSS Receiver Performance under Multipath, Ionospheric, and Interference Conditions

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Abstract

Global Navigation Satellite Systems are foundational to timing, positioning, and navigation across consumer and professional applications. Their signals traverse long propagation paths and arrive at receivers that operate with constrained bandwidth, dynamic motion, and limited computational resources. In such conditions, the performance of a receiver is shaped by three dominant impairments: reflections that distort the code correlation shape, ionospheric plasma that induces group delay and rapid phase fluctuations, and man-made radiofrequency interference that raises noise floors or mimics authentic signals. This paper presents a comprehensive evaluation of receiver behavior under these concurrent stressors with emphasis on model fidelity, algorithmic robustness, and measurement repeatability. The study organizes a signal and observation framework that spans multi-constellation, multi-frequency operation and implements canonical estimators alongside modern robust and adaptive variants. Controlled channel emulation and field recordings are used to examine code and carrier tracking, time-transfer stability, integrity risk, and resilience to deceptive waveforms. Emphasis is placed on disentangling confounding effects so that degradations in positioning and timing are attributable to physically interpretable mechanisms rather than implementation details alone. The analysis highlights the relative contributions of specular and diffuse multipath, storm-time ionospheric disturbances, and narrowband as well as wideband interferers to the observed error budgets. The results are summarized in terms of practically relevant metrics including lock probability, protection levels, and outage likelihood across a range of dynamics and antenna configurations. The findings aim to support principled choices in receiver design and deployment without overstating the generality of the conclusions.

1. Introduction

The engineering of satellite-based positioning and timing receivers amounts to orchestrating weak spread-spectrum signals, time-varying propagation, and finite-precision computations into reliable measurements (1). Received powers are far below the thermal noise floor prior to correlation, the observable structure is imprinted by ranging codes and carriers that are susceptible to channel dispersion, and the receiver front-end imposes bandwidth constraints that regulate both noise and distortion. It is therefore unsurprising that performance emerges not from a single dominant factor but from the interplay of multipath, ionosphere, and interference, each of which acts over different temporal and spectral scales. The central objective here is to examine the response of representative tracking and estimation algorithms when those phenomena overlap rather than occur in isolation, because deployments in urban, maritime, and high-latitude environments rarely present a single clean impairment class.

In designing such receivers, engineers must negotiate a balance between sensitivity and robustness. A receiver that is too narrowly optimized for thermal noise suppression may become fragile when exposed to dynamic interference or non-line-of-sight conditions, whereas one that prioritizes interference resilience may suffer from degraded accuracy under nominal conditions. The underlying mathematics, dominated by correlation and estimation theory, reflects this trade-off. The receiver correlator operates as a matched filter that seeks alignment between the incoming signal and locally generated replicas of the spreading code and carrier. Its success depends on maintaining phase coherence and code alignment within a few degrees or chips, even though the signal-to-noise ratio per sample is often below −20 dB (2). Consequently, much of the receiver's capability derives from coherent and non-coherent integration over time, which allows faint signals to be accumulated into statistically significant estimates. However, such integration presupposes stable propagation and oscillator behavior, conditions rarely satisfied in practice.

Multipath propagation, for instance, introduces delayed and phase-shifted replicas of the desired signal that interfere constructively or destructively at the receiver front-end. When the delays are comparable to or smaller than the chip duration of the spreading code, their effects manifest as bias in the correlation function, distorting the apparent code phase and hence the measured range. This bias can vary rapidly as the receiver or environment changes, producing flickering errors that resist averaging. Mitigation strategies often rely on narrow correlator spacing, multipath estimation loops, or advanced signal processing such as vector tracking and multipath-resolving

Table 1. Main impairment mechanisms in satellite positioning and timing receivers.

Phenomenon	Origin	Effect	Time Scale	Mitigation
Multipath	Surface reflections	Correlation bias, code error	ms–s	Narrow correlator, estimation loops
Ionosphere	Plasma dispersion	Frequency delay	s–min	Dual-frequency, model correction
Interference	Emitters, jammers	Noise rise, saturation	Inst.–s	Notch, spectral or spatial filtering
Quantization	Finite precision	Nonlinear bias	Constant	AGC, dithering, bit-depth tuning

algorithms. Yet these methods are only partially successful because the multipath structure is not static—it evolves as reflections appear, disappear, and fluctuate in amplitude. The complexity of urban canyons and maritime reflections further complicates modeling, forcing practitioners to rely on empirical validation and adaptive schemes.

The ionosphere introduces a different class of distortion, one that acts primarily through frequency-dependent delay and phase advance (3). Because the ionosphere is a dispersive medium, the group delay experienced by a signal depends on its carrier frequency, leading to range errors that can reach tens of meters under disturbed conditions. Dual-frequency receivers exploit this frequency dependence to estimate and remove the first-order ionospheric delay, but residual effects from higher-order terms and temporal fluctuations persist. Moreover, single-frequency receivers—still dominant in many mass-market applications—must depend on empirical or model-based corrections such as the Klobuchar model, which offer only partial compensation. During geomagnetic storms or at high latitudes, ionospheric scintillation can induce rapid amplitude and phase fluctuations, challenging even advanced tracking loops that rely on phase-locked or frequency-locked discriminators. The result is a stochastic regime where the receiver must balance loop bandwidth, damping ratio, and integration time to maintain lock without overreacting to noise.

Interference represents yet another axis of complexity. It may arise unintentionally from other radio systems or intentionally from jamming and spoofing sources. Since satellite navigation signals are weak by design, even modest interference can saturate front-end amplifiers or desensitize correlation processes (4). Narrowband interference typically manifests as spurious spectral peaks that leak into the baseband after downconversion, while broadband interference raises the effective noise floor. Modern receivers employ a mixture of time-frequency domain filtering, adaptive notch filters, and spatial processing via antenna arrays to mitigate such effects. However, these defenses are not cost-free: they introduce latency, consume computational power, and may inadvertently distort legitimate signals. The problem becomes even more intricate when interference coexists with multipath or scintillation, as the optimal mitigation for one impairment can amplify the impact of another.

A further complication arises from the limitations of digital arithmetic. Finite-precision computations in the receiver's digital signal processing chain can introduce quantization noise and nonlinearities. For example, low-bit quantizers in high-speed front-ends must capture wide dynamic ranges without saturating under interference. Designers often employ automatic gain control (AGC) and dithering techniques to preserve linearity, but these mechanisms alter the statistical properties of the signal. When integrated with tracking loops, quantization effects can subtly bias estimates of carrier phase or code delay, particularly in weak-signal conditions where small systematic errors accumulate over long integrations (5). The overall architecture must therefore reconcile analog front-end design with digital processing constraints, ensuring that noise figures, filter responses, and word lengths are matched to the intended performance regime.

The practical environments in which satellite receivers operate exacerbate these theoretical challenges. In urban areas, dense building structures create both multipath and intermittent line-of-sight conditions, forcing receivers to rely on fragmentary or non-line-of-sight signals. Maritime environments, by contrast, produce slowly varying reflections from the sea surface, leading to oscillatory bias patterns that depend on vessel motion and sea state. High-latitude and equatorial regions are characterized by ionospheric irregularities that induce scintillation, while polar regions introduce geometric challenges due to low satellite elevation angles. Each setting imposes distinct spectral-temporal signatures on the received signal, and yet real deployments often combine these effects. A ship navigating near an Arctic harbor, for example, may experience multipath from metallic structures, interference from onboard electronics, and scintillation from auroral activity—all at once.

To assess the resilience of tracking and estimation algorithms under such composite impairments, researchers construct models that combine deterministic and stochastic elements (6). Multipath may be simulated using delayed replicas with Rayleigh or Rician amplitude statistics, while ionospheric scintillation can be modeled as phase noise with specified power spectral densities. Interference is typically represented as additive colored noise or as structured sinusoidal tones. The receiver's tracking loops—code, carrier, and delay lock loops—are then analyzed in both the time and frequency domains to quantify jitter, bias, and loss-of-lock probability. Such simulations reveal that performance is highly nonlinear: small increases in interference power or multipath delay can trigger abrupt transitions from stable tracking to complete loss of signal. This nonlinear sensitivity underscores why empirical testing remains indispensable, even with sophisticated analytical models.

A complementary perspective is offered by estimation the-

Table 2. Environmental contexts and their characteristic impairments on receiver tracking and estimation.

Environment	Dominant Effects	Dynamics	Receiver Impact	Mitigation Approach
Urban Canyon	Multipath, NLOS, interference	Fast	Range bias, flicker, lock loss	Vector tracking, adaptive loops
Maritime	Surface reflections	Slow periodic	Oscillatory code bias	Temporal averaging, multipath rejection
High-Latitude / Equatorial	Ionospheric scintillation	Medium–fast	Phase jitter, fading	Dual-frequency tracking, robust PLLs
Polar Regions	Low elevation geometry	Slow	Weak SNR, poor geometry	Diversity antennas, geometry weighting

ory. From this viewpoint, the receiver acts as a real-time estimator of parameters such as code phase, carrier phase, and Doppler frequency, based on noisy and possibly biased observations. The Cramér–Rao lower bound provides a theoretical floor on the achievable variance of unbiased estimators, offering a benchmark against which practical algorithms can be compared. In ideal conditions, well-designed receivers approach this bound through optimal weighting and filtering (7). Under real-world impairments, however, bias dominates variance, and the effective information content of the signal declines. Algorithms such as the extended Kalman filter or particle filter attempt to maintain robustness by fusing information across channels and epochs, using models of satellite motion and receiver dynamics to stabilize estimates. Yet even these approaches falter when the underlying assumptions of independence or Gaussian noise are violated by strong multipath or nonstationary interference.

The convergence of all these phenomena makes satellite receiver engineering as much an art as a science. It requires not only mastery of signal theory and statistics but also an appreciation for environmental variability and hardware constraints. Field testing remains an essential complement to analysis, providing empirical data that expose the gaps between theoretical models and practical outcomes. Engineers must interpret such data to refine algorithms, redesign front-ends, and adjust processing parameters. The resulting systems embody a dynamic equilibrium between complexity and reliability—complex enough to handle the unpredictable interplay of impairments, yet streamlined enough to operate in real time on constrained hardware. (8)

A premise of the evaluation is that conclusions should be traceable to explicit assumptions in the signal and observation model. The baseband representation establishes how code, carrier, navigation data, and front-end characteristics map into correlator outputs, and the channel model specifies how specular reflections, diffuse scattering, plasma dispersion, and interfering emitters imprint phase, amplitude, and delay structure. Estimators for delay, Doppler, and phase are then studied under constrained integration times and dynamic stress representative of low-cost and survey-grade receivers. Throughout, we focus on algorithmic mechanisms, such as loop noise bandwidth and discriminator symmetry, that translate physical perturbations into biases, cycle slips, and protection levels.

The analysis proceeds by building a layered set of models and metrics. A signal and observation framework quantifies the correlator domain where tracking decisions are made; a multipath characterization isolates geometry-dependent biases and short-delay interference with the auto-correlation function; an ionospheric section addresses group delay removal, scintillation-induced fading, and phase screen dynamics; an interference section formulates detection and mitigation in both temporal and spatial domains; a receiver estimation section integrates raw observables into position, velocity, time, and integrity metrics using both classical and robust filters; a benchmarking section outlines repeatable experiments; and the discussion synthesizes findings, emphasizing conditions under which each mitigation is most effective. The presentation is intentionally neutral, with results described in terms of effect sizes and trade-offs rather than categorical superiority of particular methods.

2. Signal and Observation Model

A starting point for quantitative evaluation is a continuous-time complex baseband representation of the received signal at frequency band index b and satellite index k. The front-end downconverts, filters, and samples a superposition of line-of-sight components, multipath, and interference with additive thermal noise (9). A compact expression is

$$r_b(t) = \sum_{k=1}^{K} a_{k,b}(t)\, c_{k,b}\big(t - \tau_{k,b}(t)\big)\, e^{j\left(2\pi \nu_{k,b}(t)\, t + \phi_{k,b}(t)\right)} + \sum_{m=1}^{M_b} i_{m,b}(t) + n_b(t). \quad (1)$$

where $a_{k,b}(t)$ is a slowly varying amplitude including antenna gain patterns and propagation loss, $c_{k,b}(\cdot)$ is the spreading code after front-end filtering, $\tau_{k,b}(t)$ is the composite delay, $\nu_{k,b}(t)$ is the residual Doppler after numerically controlled oscillator removal, $\phi_{k,b}(t)$ is the residual carrier phase, $i_{m,b}(t)$ denotes interfering sources, and $n_b(t)$ is zero-mean proper complex Gaussian noise shaped by the equivalent noise bandwidth B_e.

A code and carrier tracking channel forms early, prompt, and late correlations by mixing with locally generated replicas $c_{k,b}(t - \hat{\tau})$ and $e^{-j2\pi\hat{\nu}t - j\hat{\phi}}$ followed by integration over T_I. Let $z_{E,P,L}$ denote the complex correlator outputs at code lags $\{-\Delta, 0, +\Delta\}$. For a single line-of-sight path and negligible data modulation within T_I, a first-order approximation gives

$$z_u \approx A\, R_c(\epsilon_u)\, e^{j\psi} + w_u, \quad u \in \{E, P, L\},$$

where A is an effective amplitude, $R_c(\cdot)$ is the filtered code auto-correlation, $\epsilon_u = \tau - \hat{\tau} + \delta_u$ with $\delta_u \in \{-\Delta, 0, +\Delta\}$, $\psi = \phi -$

Table 3. Analytical and empirical viewpoints guiding receiver design and validation.

Perspective	Focus Quantity	Ideal Behavior	Limiting Factor	Common Remedy
Correlation Theory	Code/Carrier Alignment	Matched filter optimum	Multipath bias	Narrow spacing, multipath loops
Estimation Theory	Variance $\to$ CRLB	Near-bound performance	Bias from nonidealities	Bias modeling, robust filtering
Simulation	Jitter, loss-of-lock	Smooth nonlinear transitions	Threshold effects	Monte Carlo, stress testing
Field Testing	Real composite effects	Validation of models	Environment unpredictability	Empirical calibration, parameter tuning

Table 4. Principal variables and parameters in the baseband signal and observation model.

Symbol	Meaning	Typical Range / Units	Notes
$a_{k,b}(t)$	Amplitude (including gain, loss)	–10 to 0 dB	Varies with geometry and antenna pattern
$\tau_{k,b}(t)$	Signal delay	0–70 ms	Includes geometric and ionospheric components
$\nu_{k,b}(t)$	Residual Doppler	±10 kHz	After NCO removal; depends on receiver motion
$\phi_{k,b}(t)$	Residual carrier phase	[0, 2) rad	Drives carrier discriminator
B_e	Equivalent noise bandwidth	1–20 MHz	Set by front-end filtering
T_I	Integration time	1–20 ms	Governs coherent accumulation
B_n	Loop noise bandwidth	0.5–25 Hz	Key trade-off between noise and dynamics
G_c, G_ϕ	Discriminator gains	dimensionless	Define loop sensitivity

$\hat{\phi}+2\pi(\nu-\hat{\nu})t_0$ for a representative time within the integration, and w_u are complex Gaussian with covariance governed by N_0 and B_e/T_I. In the presence of L multipath components with relative delays $\Delta\tau_\ell$ and phases θ_ℓ, the correlator vector $z = [z_E, z_P, z_L]^T$ becomes a coherent sum

$$z \approx \sum_{\ell=0}^{L} A_\ell \, \mathbf{r}(\epsilon + \Delta\tau_\ell, \theta_\ell) + w,$$

$$\mathbf{r}(\epsilon,\theta) = \left[R_c(\epsilon-\Delta)\, e^{j\theta},\; R_c(\epsilon)\, e^{j\theta},\; R_c(\epsilon+\Delta)\, e^{j\theta}\right]^T. \quad (2)$$

A delay discriminator $D_c(z)$ and a carrier discriminator $D_\phi(z)$ map z into code and phase errors. For a symmetric early-minus-late power discriminator with spacing Δ, an equivalent small-error gain G_c can be defined via

$$\mathbb{E}[D_c(z)] \approx G_c\,(\tau-\hat{\tau}) + b_c, \quad \mathbb{E}[D_\phi(z)] \approx G_\phi\,(\phi-\hat{\phi}) + b_\phi,$$

where b_c and b_ϕ represent biases due to asymmetry induced by multipath and front-end filtering. Loop dynamics are described by discrete-time stochastic difference equations, such as a second-order delay lock loop with noise and dynamic stress error (10)

$$\epsilon_{n+1} = \epsilon_n + T\,\dot{\epsilon}_n, \qquad \dot{\epsilon}_{n+1} = \dot{\epsilon}_n - \omega_n^2 T\,\epsilon_n - 2\zeta\omega_n\,\dot{\epsilon}_n + \eta_n,$$

where ω_n is the natural frequency set by the loop noise bandwidth B_n, ζ is damping, T is the update interval, and η_n collects effective measurement noise mapped through the discriminator slope. The carrier loop admits an analogous representation with process terms driven by jerk and jerk-rate for dynamic stress analysis.

Pseudorange, carrier-phase, and Doppler observables are formed from stabilized code delay, accumulated carrier, and frequency estimates. For band b the pseudorange $\rho_{k,b}$ and carrier phase $\Phi_{k,b}$ can be written as

$$\rho_{k,b} = |0\mathbf{x}_r - \mathbf{x}_k|0 + c\,(\delta t_r - \delta t_k) + T + I_b + \epsilon_\rho,$$
$$\Phi_{k,b} = |0\mathbf{x}_r - \mathbf{x}_k|0 + c\,(\delta t_r - \delta t_k) + T - I_b + \lambda_b N_{k,b} + \epsilon_\Phi. \quad (3)$$

where $\mathbf{x}_r$ and $\mathbf{x}_k$ are receiver and satellite positions at the appropriate transmission and reception times, δt_r and δt_k are clock offsets, T is the neutral atmosphere delay, I_b is the ionospheric group delay with opposite sign on phase, λ_b is the wavelength, $N_{k,b}$ is the integer ambiguity, and $\epsilon_\rho, \epsilon_\Phi$ are residual errors that include multipath, interference-induced biases, and tracking noise.

A compact observation stacking for B bands and K satellites yields

$$y = h(\mathbf{s}) + \nu, \quad \mathbf{s} = [\mathbf{x}_r, \mathbf{v}_r, \delta t_r, \dot{\delta t}_r, T, \{I_b\}_{b=1}^{B}, \{N_{k,b}\}, \ldots]^T,$$

with $h(\cdot)$ differentiable almost everywhere in the region of interest and ν a composite noise capturing colored correlator noise and bias terms. This representation enables linearization for filter-based estimation while preserving structured components for integrity analysis.

3. Characterization of Multipath and Urban Propagation

Multipath modifies correlation shapes through coherent addition of delayed replicas, and the structure depends on geometry, surface electrical properties, antenna gain, and front-end bandwidth. A widely used abstraction is a specular-diffuse decomposition in which one or more deterministic specular components are superimposed on a stochastic diffuse tail. If the strongest specular reflection has relative amplitude α and

Table 5. Characterization of multipath mechanisms and mitigation approaches in urban propagation.

Component / Model	Description	Dominant Parameters	Effect on Receiver	Mitigation Strategy
Specular Reflection	Single delayed replica	$\alpha, \Delta\tau, \theta$	Code bias, carrier ripple	Narrow correlator, multipath loops
Diffuse Tail	Random scatter	σ_μ^2, τ_0	Noise-like variance inflation	Bandwidth shaping, averaging
Multipath Envelope	Bias amplitude vs. delay	Δ, G_c	Determines worst-case bias	Optimize Δ for SNR vs. bias
Spatial Multipath	Array response $a(\theta, \varphi)$	σ_p^2, N	Directional interference	Adaptive beamforming, nulling
Urban Environment	Dynamic reflections	–	Flicker bias, loss of lock	Vector tracking, real-time adaptation

delay $\Delta\tau$ with phase θ, the prompt correlator perturbation is well approximated by

$$z_P \approx AR_c(\epsilon) + A\,\alpha\,R_c(\epsilon - \Delta\tau)\,e^{j\theta} + w,$$

which induces a code tracking bias through the discriminator nonlinearity. For early-minus-late power with spacing Δ, a small-error bias approximation is

$$b_c \approx \frac{\alpha \sin\theta}{G_c}\left[R_c'(-\Delta)\,R_c(-\Delta - \Delta\tau) - R_c'(+\Delta)\,R_c(+\Delta - \Delta\tau)\right],$$

where $R_c'(\cdot)$ is the derivative of the filtered correlation. The sign of b_c oscillates with θ and varies with $\Delta\tau$, giving bias envelopes that peak near code chip fractions determined by the front-end bandwidth and code family.

Diffuse multipath is often modeled as a zero-mean complex Gaussian process with delay power density $S(\tau)$ truncated by environment geometry. The resulting correlator covariance can be integrated analytically when $S(\tau)$ is exponential or power-law. For an exponential profile $S(\tau) = \sigma_\mu^2 \exp(-\tau/\tau_0)\mathbb{I}\{\tau \geq 0\}$, the additional prompt variance is

$$\mathrm{Var}[z_P]_{\mathrm{diffuse}} \approx \sigma_\mu^2 \int_0^\infty |R_c(\epsilon - \tau)|^2 e^{-\tau/\tau_0}\, d\tau,$$

revealing how front-end bandwidth and correlator spacing regulate the effective noise.

The multipath error envelope for code is typically quoted as a worst-case bias over θ (11). For a single reflection and symmetric R_c, the envelope becomes

$$\begin{aligned} E_{\mathrm{mp}}(\Delta\tau) &= \max_{\theta\in[0,2\pi)} \left|\hat{\tau} - \tau\right| \\ &\approx \left|\frac{\alpha}{G_c}\left[R_c'(-\Delta)\,R_c(-\Delta - \Delta\tau) - R_c'(+\Delta)\,R_c(+\Delta - \Delta\tau)\right]\right|. \end{aligned} \tag{4}$$

Different correlator spacings alter the envelope, and narrow spacing is known to reduce biases for moderate $\Delta\tau$ at the cost of increased thermal noise amplification.

An important observation is that carrier tracking error can remain small while code tracking exhibits pronounced bias, especially when reflections have delays longer than several chips so that their effect on the narrowband carrier replica is limited. The carrier phase, however, becomes fragile when multipath amplitudes approach the line-of-sight level and $\Delta\tau$ is less than a few cycles, creating phase scintillation-like behavior that interacts with Doppler dynamics.

Spatial processing with controlled reception pattern antennas introduces an additional term in the received model: an array steering vector $\mathbf{a}(\theta, \varphi)$ multiplies each path and allows adaptive beamforming and nulling. For an N-element array receiving P plane-wave components, the spatial covariance is

$$\mathbf{R} = \mathbb{E}[\mathbf{r}\mathbf{r}^H] = \sum_{p=1}^{P} \sigma_p^2 \mathbf{a}(\theta_p, \varphi_p)\mathbf{a}^H(\theta_p, \varphi_p) + \sigma_n^2 \mathbf{I},$$

which supports eigen-structure methods to separate dominant specular reflections from diffuse backgrounds. Beam-steered correlators exploit weight vectors $\mathbf{w}$ to form $z = \mathbf{w}^H\mathbf{r}$, and weights are chosen to optimize a criterion such as maximum signal-to-interference-plus-noise ratio subject to unity gain in the desired direction.

4. Ionospheric Effects and Mitigation

The ionosphere contributes a dispersive delay due to free electrons, adding positive group delay to code and negative phase advance to carrier. A standard scalar model relates slant total electron content TEC to delay on frequency f via

$$I(f) = \frac{40.3\,\mathrm{TEC}}{f^2},$$

where $I(f)$ is expressed in meters when TEC is in TECU and f in Hz. Dual-frequency combinations can remove the first-order term. The ionosphere-free code and phase combinations for bands f_1, f_2 are

$$\rho_{\mathrm{IF}} = \frac{f_1^2}{f_1^2 - f_2^2}\rho_1 - \frac{f_2^2}{f_1^2 - f_2^2}\rho_2, \qquad \Phi_{\mathrm{IF}} = \frac{f_1^2}{f_1^2 - f_2^2}\Phi_1 - \frac{f_2^2}{f_1^2 - f_2^2}\Phi_2, \tag{12}$$

which increase measurement noise while suppressing first-order dispersion (13). Alternatively, geometry-free combinations isolate ionospheric variations for estimation and quality control.

During disturbed conditions, amplitude and phase scintillation arise from propagation through irregularities. A common abstraction is a phase screen with structure function $D_\phi(\tau) = C_\phi \tau^\beta$ over a temporal inertial range. The effect on a phase-locked loop with noise bandwidth B_n and integration time T_I can be approximated by an equivalent process

Table 6. Summary of ionospheric delay phenomena, their impacts, and common mitigation techniques.

Effect / Model	Mathematical Relation	Impact	Mitigation Method	Residual Error
First-Order Dispersion	$I(f) = 40.3\,\mathrm{TEC}/f^2$	Code delay, phase advance	Dual-frequency or model correction	1–10 m (single freq.)
Ionosphere-Free Combo	Weighted ρ_1, ρ_2 or Φ_1, Φ_2	Noise amplification	Linear combination	$\approx 3\times$ noise increase
Scintillation (Phase)	$D_\phi(\tau) = C_\phi \tau^\beta$	Phase jitter, slips	Adaptive PLL bandwidth, loss detection	$P_{\text{slip}} \sim e^{-\pi^2/(8\sigma_\phi^2)}$
State-Space Delay Estimation	$\dot{x}_k = \omega_k$	Temporal TEC variation	Kalman filtering w/ elevation mapping	cm–dm residuals
Higher-Order Terms	$I(f) = 40.3/f^2 + C_2/f^3 + C_3/f^4$	Bias at high precision	Include magnetic terms	< 1 cm at L-band

noise spectral density $S_\phi(\omega) \propto \omega^{-(1+\beta)}$ mapped into the loop via its sensitivity function $H_\phi(e^{j\omega T_I})$. The cycle-slip probability for a second-order loop can then be related to the standardized phase error variance σ_ϕ^2 as

$$P_{\text{slip}} \approx \exp\left(-\frac{\pi^2}{8\sigma_\phi^2}\right),$$

which emphasizes the compounding effect of scintillation-driven process noise and reduced C/N_0.

A state-space approach estimates ionospheric delay along each line-of-sight. Let $x_k(t)$ denote a slant delay state projected from a thin-shell model with mapping function $M(e)$ depending on elevation e. With random-walk dynamics $\dot{x}_k = \omega_k$, the code measurement on band b can be written as

$$\rho_{k,b} = \ldots + \gamma_b x_k + \varepsilon_{k,b}, \qquad \gamma_b = \frac{40.3}{f_b^2},$$

with ellipsis denoting geometric and clock terms. Stacking satellites allows joint estimation of receiver clock and ionospheric states via a Kalman filter that uses process noise tuned to diurnal variability or enhanced during storm warnings to accommodate increased dynamics. (14)

Higher-order ionospheric effects, notably the frequency-squared and frequency-cubed terms due to Earth's magnetic field and electron density gradients, can be incorporated when centimeter-level accuracy is targeted. A perturbation expansion yields

$$I(f) \approx \frac{40.3\,\mathrm{TEC}}{f^2} + \frac{C_2}{f^3} + \frac{C_3}{f^4},$$

where coefficients C_2, C_3 depend on path geometry and geomagnetic parameters. While typically small at L-band, their residual effect may matter when dual-frequency combinations are used in high-precision modes with long integrations.

5. Interference, Jamming, and Spoofing Analysis

Interference is represented as an additive process superposed on the authentic signal. A parametric form that spans narrowband continuous-wave, multi-tone, and wideband noise-like interferers is

$$i(t) = \sum_{q=1}^{Q} A_q\, e^{j(2\pi f_q t + \phi_q)} + (g * w_J)(t),$$

where f_q are tone offsets, g is a shaping filter, and w_J is Gaussian white noise scaled to achieve a jammer-to-signal ratio. After correlation, narrowband components leak through as slowly varying complex biases on correlator outputs, while wideband noise increases variance uniformly. The effective post-correlation interference-to-noise ratio is (15)

$$\mathrm{INR}_{\text{pc}} \approx \frac{P_I T_I}{N_0 B_e},$$

with P_I the in-band interference power.

Detection strategies derive from binary hypothesis tests. For a correlator vector z with known noise covariance ∘ and an interference subspace spanned by $\mathbf{H}$, a generalized likelihood ratio test reduces to a quadratic form

$$\Lambda(z) = z^{H\circ-1}\mathbf{H}\left(\mathbf{H}^{H\circ-1}\mathbf{H}\right)^{-1}\mathbf{H}^{H\circ-1}z \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \eta,$$

where the threshold η is set by a chosen false alarm rate. Time-frequency detectors supplement this with energy concentration measures obtained from short-time Fourier transforms of raw or intermediate frequency samples prior to correlation.

Spoofing introduces structured replicas synchronized near authentic signals. A simple single-tap spoofer at relative code delay δ and Doppler $\Delta\nu$ yields post-correlation statistics that deviate from the symmetry expected under multipath. A vector of early, prompt, and late products exposes asymmetry through signed consistency checks. One such check uses a signed area (16)

$$S = \mathrm{Re}\{z_E z_P^*\} - \mathrm{Re}\{z_L z_P^*\},$$

whose expectation is near zero for symmetric distortions but grows in magnitude for asymmetric pull-off maneuvers. Spatial processing with arrays extends detection by examining eigenvalue spreads of the covariance matrix, as spoofers often arrive from distinct directions relative to the diffuse environment. For an N-element array, the sphericity test

$$T = \frac{\left(\prod_{i=1}^{N} \lambda_i\right)^{1/N}}{\frac{1}{N}\sum_{i=1}^{N} \lambda_i}$$

departs from unity as strong directional components dominate, providing a feature for change detection.

Adaptive mitigation tunes front-end notch filters for tones and applies time-varying weights in array beamformers to

Table 7. Representative interference and spoofing forms, their parameters, receiver impacts, and canonical detection or mitigation strategies.

Type	Mathematical Representation	Dominant Parameters	Receiver Impact	Detection / Mitigation
Narrowband CW	$Ae^{j(2\pi ft+\phi)}$	Tone offset f, amplitude A	Correlator bias, periodic jitter	Adaptive notch, spectral excision
Wideband Noise	$(g * w_J)(t)$	Bandwidth B_J, power P_I	Raised noise floor, reduced C/N_0	Filtering, AGC optimization
Multi-Tone	$\sum_q A_q e^{j(2\pi f_q t+\phi_q)}$	$\{f_q\}, \{A_q\}$	Correlator distortion, loss of lock	Notch filtering, robust loops
Spoofing	Delayed replica with $(\delta, \Delta\nu)$	Relative delay/Doppler	Asymmetric correlations, pull-off	Symmetry checks, direction finding
Array-Based	Structured field $\mathbf{a}(\theta, \varphi)$	DOA, covariance $\mathbf{R}$	Spatial bias, phase inconsistency	LCMV beamforming, sphericity test

maximize carrier-to-noise ratio while preserving code correlation symmetry. A linearly constrained minimum variance solution with constraint vector $\mathbf{c}$ and interference-plus-noise covariance $\mathbf{R}$ is

$$\mathbf{w}_{\mathrm{LCMV}} = \frac{\mathbf{R}^{-1}\mathbf{c}}{\mathbf{c}^H\mathbf{R}^{-1}\mathbf{c}},$$

delivering unit gain toward the desired direction and attenuation elsewhere. In scalar receivers with no spatial freedom, adaptive whitening and frequency-domain excision reduce narrowband interference with modest distortion of the correlation function if the notches are sufficiently narrow compared to the code spectrum.

6. Multi-Tone Interference and Receiver Response

Multi-tone interference arises when two or more narrowband emitters occupy spectral locations within or close to the receiver's front-end passband, generating deterministic spectral lines that persist through downconversion and sampling. Unlike wideband noise, which raises the variance floor uniformly, a cluster of tones introduces structured bias and periodicity into the correlator outputs that interact with tracking discriminators in ways that depend on tone placement, amplitude, phase, and loop settings (17). The phenomenology is further shaped by the front-end bandwidth, the automatic gain control dynamics, and the integration time chosen by the receiver. Experimental campaigns across varied environments have documented that closely spaced tones can precipitate loss of lock, anomalous group delay estimates, and spurious integrity flags. As one example among many, experimental work has reported that multi-tone interference can cause substantial degradations in receiver performance, including the results reported by Tsintsadze et al. (2024), but such observations are consistent with broader analytical expectations derived from correlator-domain models and loop stability analyses (18). The purpose of this section is to examine the mechanisms through which multi-tone interference alters observables and to outline mitigation approaches that attenuate those mechanisms without presuming a single configuration or declaring a universal remedy.

A convenient starting point is a complex baseband representation in which the useful spread-spectrum signal coexists with a superposition of tones. Writing the interfering component as a sum of Q complex sinusoids with offsets $\{f_q\}$ relative to the local carrier replica and amplitudes $\{A_q\}$,

$$i(t) = \sum_{q=1}^{Q} A_q\, e^{j(2\pi f_q t+\phi_q)},$$

one observes that correlation with a locally generated code replica over an integration interval T_I effectively filters each tone by a sinc-like kernel centered at the residual Doppler ν and scaled by front-end responses. If z denotes the complex prompt correlator output and $G(\cdot)$ the equivalent frequency response of the integration and front-end cascade, the interference contribution is approximated by (19)

$$z_I \approx \sum_{q=1}^{Q} A_q\, G(\nu - f_q)\, e^{j(\phi_q - \hat{\phi})},$$

where $\hat{\phi}$ is the receiver's carrier replica phase. The magnitude $|G(\cdot)|$ peaks when the tone offset aligns with the residual Doppler, but even for offsets substantially different from ν the sidelobes of G introduce leakage that appears as a slowly varying complex bias at the correlator output. Since code and carrier discriminators rely on symmetry and phase consistency across early, prompt, and late taps, the presence of such a bias perturbs discriminator outputs in a manner that depends on tone phases and relative placements.

The net impact on tracking can be summarized through the effective post-correlation interference-to-noise ratio,

$$\mathrm{INR}_{\mathrm{pc}} = \frac{1}{N_0 B_e} \sum_{q=1}^{Q} |A_q|^2\, |G(\nu - f_q)|^2\, T_I,$$

which quantifies the variance contribution of tones relative to thermal noise over the integration interval. However, variance alone underrepresents the effect of tones, because the complex offset z_I induces deterministic distortions in discriminator characteristics. Consider an early-minus-late power discriminator with spacing Δ and prompt output z_P. Writing early and late outputs as $z_E = z_P + \delta_E$ and $z_L = z_P + \delta_L$ with small code errors, the discriminator's expected value acquires a bias term b_c that is approximately linear in $\mathrm{Re}\{z_I z_P^*\}$ and its analogs for early and late taps. The resulting small-error model,

$$\mathbb{E}[D_c] \approx G_c\,(\tau - \hat{\tau}) + b_c, \qquad b_c \propto \mathrm{Re}\{z_{I,E} z_P^* - z_{I,L} z_P^*\},$$

Table 8. Principal estimation, tracking, and integrity formulations with their associated roles, biases, and mitigation strategies.

Function	Model / Expression	Role	Error Source / Bias Term	Mitigation or Metric
State Estimation	$\mathbf{x}_{k+1} = \mathbf{F}\mathbf{x}_k + \mathbf{G}\mathbf{w}_k$	Propagate dynamics	Process noise, clock drift	Process tuning, inertial aiding
Measurement Model	$\mathbf{y}_k = \mathbf{H}_k\mathbf{x}_k + \mathbf{d}_k + \mathbf{v}_k$	Link to observables	Multipath, ionosphere	Bias states, dual-frequency correction
Robust Filtering	$\Delta\hat{\mathbf{x}} = (\mathbf{H}^T\mathbf{W}\mathbf{H})^{-1}\mathbf{H}^T\mathbf{W}\mathbf{r}$	Downweight outliers	Non-Gaussian residuals	M-estimation (Huber, Tukey)
Integrity Level	$\mathrm{PL} = K_\alpha\sqrt{\mathbf{g}^T(\mathbf{H}^T\mathbf{R}^{-1}\mathbf{H})^{-1}\mathbf{g}} + \delta_{\text{bias}}$	Risk bounding	Unmodeled bias	Conservative protection levels
Consistency Check	$\pi = \lvert 0\mathbf{S}\mathbf{y}\rvert 0$	Fault detection	Correlated errors	Threshold via false alarm allocation
Tracking Jitter	$\sigma_\tau^2 = \frac{B_n}{2C/N_0}\frac{1}{G_c^2 T_I} + \sigma_{\text{mp}}^2 + \sigma_{\text{iono}}^2 + \sigma_{\text{int}}^2$	Code loop noise prediction	Multipath, interference	Optimize B_n, adaptive T_I

illustrates how tone-induced complex offsets, which may differ slightly across early, prompt, and late lags due to code-spectrum filtering, translate into apparent code error even when the true delay is centered. The sign and magnitude of b_c oscillate with the phases $\{\phi_q\}$ and evolve as residual Doppler drifts, leading to slowly varying biases superposed on thermal jitter. When multiple tones are present, cross-terms between tones at separations commensurate with the inverse of T_I generate beating that appears as periodic excursions in discriminator outputs. (20)

Carrier tracking is similarly affected. A second-order phase-locked loop with natural frequency ω_n and damping ζ can be described by a phase error state θ driven by a nonlinear restoring term and disturbances. The presence of a tone at offset f_q yields an additive periodic torque term in the loop error dynamics,

$$\ddot{\theta} + 2\zeta\omega_n\dot{\theta} + \omega_n^2 \sin\theta = \sum_{q=1}^{Q} \kappa_q \cos(2\pi f_q t + \psi_q) + \xi(t),$$

where $\xi(t)$ aggregates thermal noise and any residual process noise from dynamics, and κ_q depends on A_q, $G(\nu - f_q)$, and discriminator sensitivity. Describing-function analysis linearizes $\sin\theta$ for small errors and reveals that tones near the loop bandwidth inject quasi-resonant drive that amplifies phase excursions. As the aggregate periodic drive grows, the loop exhibits increased phase jitter and, beyond a threshold, slips. The slip probability over an interval, while contingent on the detailed noise distribution and tone phases, increases with the effective variance of the driven error process. An approximate expression of the form (21)

$$P_{\text{slip}} \approx \exp\left(-\frac{\Theta^2}{2\sigma_\theta^2}\right),$$

with Θ a critical phase excursion and σ_θ^2 an effective variance combining stochastic and periodic contributions, illustrates the exponential sensitivity of cycle continuity to tone energy that happens to align with loop sensitivity.

The automatic gain control (AGC) modifies the picture by compressing the front-end response to maintain nominal levels. If the AGC time constant is comparable to the coherent integration time, amplitude fluctuations induced by beating between tones can translate into epoch-to-epoch scaling of correlator outputs. Because discriminators typically normalize by prompt magnitudes or use power differences, such scaling appears in higher-order terms that couple with multipath and residual frequency errors. A simplified scalar model in which the AGC gain $g(t)$ follows a first-order response to the envelope power $p(t)$,

$$\dot{g}(t) = -\frac{1}{\tau_g}\big(g(t) - g_0\big) - k_g\big(p(t) - p_0\big),$$

with τ_g the time constant, g_0 the nominal gain, and p_0 a target power, shows that multi-tone envelopes with slow beats relative to τ_g induce quasi-static gain deviations that bias correlator magnitudes across early, prompt, and late taps. If τ_g is short relative to the highest beat frequency, the AGC follows the envelope rapidly, imprinting additional amplitude modulation that converts to nonstationary noise after correlation.

Mitigation strategies fall broadly into spectral excision, time-varying adaptive cancellation, spatial filtering, and estimator-level robustness. In scalar receivers that cannot exploit spatial diversity, notch filtering is the canonical response (22). A digital notch with zeros aligned to the tone frequencies attenuates their energy while minimizing collateral damage to the code spectrum if the notches are narrow. A two-pole, two-zero section that realizes a notch at normalized radian frequency ω_q with pole radius $r < 1$ is

$$H_q(z) = \frac{1 - 2\cos\omega_q\, z^{-1} + z^{-2}}{1 - 2r\cos\omega_q\, z^{-1} + r^2 z^{-2}},$$

and the composite filter for Q tones is $H(z) = \prod_{q=1}^{Q} H_q(z)$. The notch depth and width are governed by r; values close to unity realize deep, narrow notches that preserve most of the spread-spectrum energy but increase group-delay ripple, which in turn can distort the correlation shape modestly. The distortion is acceptable when notches are sparse and well separated from the code spectrum's main lobes, but when tones cluster near correlation-sensitive bands the trade-offs become acute.

Frequency-domain excision operates on short-time Fourier transforms of the intermediate-frequency (IF) or baseband sequence. Windowed segments are transformed, bins exceeding adaptive thresholds are attenuated, and inverse transforms reconstruct the time series. If the window length is N and the sampling rate f_s, a bin width of f_s/N must be chosen fine enough to isolate tones yet coarse enough to limit latency and computational load. The excision operator can be written as

$$x[n] = \mathcal{F}^{-1}\{W[k] \cdot \mathcal{F}\{r[n]\}[k]\}, \qquad W[k] = \begin{cases} \alpha, & |X[k]| > \eta \\ 1, & \text{otherwise}, \end{cases}$$

where $\alpha \in [0, 1)$ attenuates contaminated bins and η is a threshold set relative to median or robust estimates of spectral magnitude (23). This approach spreads notch distortion over the window, often resulting in milder correlation shape perturbations than sharp IIR notches, at the expense of block processing artifacts if windowing and overlap-add parameters are not carefully tuned.

Spatial filtering with controlled reception pattern antennas or arrays adds another dimension of mitigation by placing nulls in the directions of interfering emitters while maintaining gain toward satellites. If $\mathbf{a}_s$ denotes the array steering vector toward the desired signal and $\{\mathbf{a}_q\}$ those toward tones, a linearly constrained minimum variance beamformer selects weights

$$\mathbf{w} = \arg\min_{\mathbf{w}} \mathbf{w}^H \mathbf{R}\, \mathbf{w} \quad \text{subject to} \quad \mathbf{C}^H \mathbf{w} = \mathbf{f},$$

where $\mathbf{R}$ is the sample covariance matrix, $\mathbf{C} = [\,\mathbf{a}_s, \mathbf{a}_1, \dots, \mathbf{a}_Q\,]$ encodes constraints, and $\mathbf{f} = [\,1, 0, \dots, 0\,]^T$ enforces unity response toward the satellite and nulls toward tones. The closed-form solution,

$$\mathbf{w} = \mathbf{R}^{-1}\mathbf{C}\left(\mathbf{C}^H \mathbf{R}^{-1} \mathbf{C}\right)^{-1} \mathbf{f},$$

reweights array elements to suppress tones with minimal increase in noise. In practice, imperfect knowledge of $\mathbf{a}_q$ and time variation in the interference field necessitate adaptive updates and diagonal loading to stabilize inversions in finite-sample regimes.

Detection precedes mitigation in many designs, providing timely alerts when tone energy rises or appears in critical bands. Energy detectors operate on raw or pre-correlation spectra, comparing bin magnitudes to robust baselines. A simple generalized likelihood ratio test treats a set of bins $\mathcal{K}$ as a potential tone subspace and evaluates

$$\Lambda = \sum_{k \in \mathcal{K}} \frac{|X[k]|^2}{\hat{\sigma}^2[k]} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \eta,$$

with $\hat{\sigma}^2[k]$ an estimate of nominal bin variance and η a threshold chosen to meet a false alarm target over the number of independent bins examined. In the correlator domain, asymmetry checks that compare normalized early and late products reveal departures from the symmetry expected of noise-limited conditions. Defining a signed statistic such as $S = \mathrm{Re}\{z_E z_P^*\} - \mathrm{Re}\{z_L z_P^*\}$ and monitoring $\{|S|\}$ across channels offers a low-cost indicator that a structured bias has arisen.

Estimation-level techniques complement front-end measures by reducing sensitivity to contaminated observables (24). Robust loss functions within least-squares or filter updates downweight epochs and satellites exhibiting residuals inconsistent with nominal covariance models. If $\mathbf{r}$ denotes the innovation vector and $\mathbf{R}$ its covariance, a weighted update with componentwise weights $w_i = \psi(r_i/\sigma_i)/(r_i/\sigma_i)$ yields

$$\Delta\hat{\mathbf{x}} = \left(\mathbf{H}^T \mathbf{W} \mathbf{H}\right)^{-1} \mathbf{H}^T \mathbf{W}\, \mathbf{r}, \qquad \mathbf{W} = \mathrm{diag}(w_1, \dots, w_n),$$

where $\psi(\cdot)$ is chosen to bound influence for large standardized residuals. When tones induce biases that manifest intermittently as large residuals on particular satellites or bands, such weighting prevents those epochs from dominating the solution while preserving efficiency when conditions are nominal. Integrity metrics that include explicit bias allowances account for the possibility that residual tone effects remain after mitigation, maintaining conservative bounds in safety-critical applications.

The alignment between tone locations and the code spectrum deserves special attention. Spread-spectrum signals concentrate energy in a main lobe around the carrier with decaying sidelobes dictated by the code's spectral properties and the front-end filter. Tones that fall near spectral regions where the code's sensitivity to correlation shape distortions is high produce outsized discriminator biases even at modest power levels. An approximate sensitivity kernel can be derived by writing the early-minus-late discriminator response as a functional of the received spectrum $S_r(f)$ and linearizing around the nominal spectrum $S_0(f)$,

$$D_c \approx D_c^{(0)} + \int_{-B}^{B} K(f)\,\big(S_r(f) - S_0(f)\big)\, df,$$

with $K(f)$ computed from the derivative of the correlation function with respect to spectral perturbations (25). The kernel peaks where small spectral alterations translate into large changes in the slope of the correlation function at the prompt. From this perspective, mitigation that succeeds in redistributing tone energy away from high-sensitivity bands achieves disproportionate benefit relative to uniform attenuation.

Integration time T_I interacts with multi-tone fields in nontrivial ways. Longer integrations sharpen $G(\cdot)$, reducing leakage from tones off the residual Doppler but increasing the risk that tones near ν imprint stronger complex biases due to narrower main lobes. Moreover, as T_I grows, the number of code transitions within an integration epoch changes, and residual navigation data or pilot-only modes alter the effective interference mapping. Adaptive integration that shortens T_I when detectors report rising tone energy near sensitive offsets can moderate correlator bias at the expense of increased thermal jitter. Such adaptation requires consistency with loop bandwidths to avoid creating dynamics-induced tracking stress.

Interactions with multipath and scintillation further complicate response (26). Multipath creates its own deterministic biases by distorting the correlation function, and tone-induced complex offsets add vectorially to those distortions. When a tone's phase happens to align to counteract a multipath-induced bias, transient improvements in performance can be observed, but the subsequent phase drift reverses the effect, producing apparent oscillations in code error. Scintillation reduces effective carrier-to-noise density ratio and introduces rapid phase fluctuations that cause tone leakage through $G(\cdot)$ to vary across epochs, modulating the amplitude of tone-induced biases and jitter. These interactions recommend detectors and mitigations that look for multi-domain consistency, combining spectral, time-domain, and correlator symmetry cues rather than relying on any single indicator.

From a design standpoint, front-end linearity and dynamic range shape the onset of nonlinear artifacts under strong tones. If a tone drives mixers or amplifiers toward saturation, intermodulation products at sums and differences of tone frequencies appear, effectively increasing Q and distributing energy into previously uncontaminated bins. While careful component selection and gain distribution postpone this regime, receivers exposed to harsh environments benefit from protective limiters and filters that constrain the maximum in-band power delivered to sensitive stages. The cost is added insertion loss and potential group-delay ripple, again illustrating the trade-offs that pervade mitigation decisions.

A recurring observation in both analysis and field practice is that simple, well-tuned measures often outperform more elaborate but brittle strategies (27). For example, a modest number of carefully placed digital notches combined with conservative robust weighting in the estimator can deliver stable performance across a wide range of tone constellations. Conversely, aggressive spectral excision with deep, broad notches can introduce sufficient correlation distortion to degrade code tracking in quiet conditions, yielding a net loss in average performance. Similarly, array processing provides dramatic improvements when tone directions are stable and well separated from satellites but may falter when emitter bearings fluctuate rapidly or when calibration drifts introduce steering errors. The pragmatic approach is to layer mitigations, selecting thresholds and adaptation rates that avoid rapid oscillations between modes and reserving invasive actions for clear detections with strong margins.

Quantitative assessment of multi-tone impacts benefits from metrics that discriminate between variance inflation and bias. Root-mean-square tracking error conflates the two; separating median absolute error from standard deviation clarifies whether mitigation reduces bias, jitter, or both. Cycle-slip rates on carrier and lock probabilities on code under tone ramps provide actionable statistics for loop design. Time-transfer stability assessed by Allan deviation reveals whether periodicity introduced by tone beating leaks into fractional frequency fluctuations at specific averaging times (28). Integrity-oriented metrics that map residual tone effects into protection level components support risk-informed choices on mitigation aggressiveness.

The cumulative picture is not that multi-tone interference invariably forces loss of navigation, but rather that it introduces a structured stressor whose effect depends on alignment with receiver sensitivities. Placement of tones near loop bandwidths and correlation-sensitive spectral regions exerts disproportionate influence relative to equal-power tones positioned benignly. Receivers that monitor those alignments in real time and adjust integration, filtering, and weighting accordingly maintain more stable performance, while receivers with fixed settings exhibit wider variability across tone constellations. Observations across many experiments, including those that have reported substantial degradations under multi-tone conditions, agree with these mechanisms without implying that any single dataset captures the full variety of scenarios likely to be encountered in practice.

Finally, it is helpful to view multi-tone mitigation in the broader context of receiver architecture evolution. The increasing prevalence of pilot channels enables longer coherences that sharpen $G(\cdot)$, which is advantageous for thermal noise but raises sensitivity to near-residual tones unless complemented by adaptive integration control. Multi-constellation, multi-frequency operation distributes risk by offering alternative measurements less affected by a given tone cluster, provided that cross-frequency coupling in the front end does not transfer contamination. Tight coupling with inertial or other aiding sensors allows loop bandwidth reduction during interference episodes, reducing susceptibility to periodic drive at the cost of increased reliance on aiding integrity (29). None of these adjustments eliminates the fundamental coupling between deterministic tones and correlator-domain symmetry, but together they narrow the conditions under which tones precipitate severe performance loss and align mitigation costs with the likelihood and impact of specific interference patterns.

7. Receiver Estimation, Tracking, and Integrity

A receiver synthesizes observables into navigation states using estimators that trade noise against bias and exploit dynamics. A common choice is an extended Kalman filter driven by code and carrier measurements that are linearized about predicted satellite positions and clocks. Let the state be $\mathbf{x} = [\mathbf{p}, \mathbf{v}, \delta t, \dot{\delta t}]^T$ with process model

$$\mathbf{x}_{k+1} = \mathbf{F}\mathbf{x}_k + \mathbf{G}\mathbf{w}_k,$$

where $\mathbf{w}_k$ aggregates acceleration and clock process noises. The measurement model stacks pseudoranges and, when available, carrier-phase double differences or ionosphere-free combinations

$$\mathbf{y}_k = \mathbf{H}_k\mathbf{x}_k + \mathbf{d}_k + \mathbf{v}_k,$$

with $\mathbf{d}_k$ capturing slowly varying biases from multipath and residual ionosphere. A bias-aware filter augments $\mathbf{x}$ with states that follow random walks at rates reflecting expected variability so that long integrations can separate geometry and bias contributions without overfitting.

Robustification is valuable when interference or scintillation produce outliers and non-Gaussian residuals. An M-estimator modifies the quadratic loss to a bounded influence function. In a Gauss–Newton iteration, the update becomes

$$\Delta\hat{\mathbf{x}} = \left(\mathbf{H}^T\mathbf{W}\mathbf{H}\right)^{-1}\mathbf{H}^T\mathbf{W}\,\mathbf{r}, \qquad \mathbf{W} = \mathrm{diag}\left\{\frac{\psi(r_i/\sigma_i)}{r_i/\sigma_i}\right\},$$

where $\psi(\cdot)$ is a robust score such as Huber or Tukey and $\mathbf{r}$ are residuals. This downweights measurements exhibiting

Table 9. Multi-tone interference mechanisms and associated mitigation strategies across spectral, temporal, and spatial domains.

Mechanism	Expression	Effect	Key Parameter	Mitigation / Trade-Off
Correlator Bias	$z_I = \sum_q A_q G(\nu - f_q) e^{j(\phi_q - \hat{\phi})}$	Deterministic bias b_c	T_I, ν	Adjust T_I, remove tones near ν
Phase Loop Drive	$\ddot{\theta} + 2\zeta\omega_n\dot{\theta} + \omega_n^2 \sin\theta = \sum_q \kappa_q \cos(2\pi f_q t)$	Phase jitter / slips	ω_n, ζ, f_q	Tune BW, apply aiding
AGC Coupling	$\dot{g} = -\tau_g^{-1}(g - g_0) - k_g(p - p_0)$	Amplitude bias	τ_g, k_g	Choose proper τ_g
Spectral Excision	$H_q(z) = \frac{1 - 2\cos\omega_q z^{-1} + z^{-2}}{1 - 2r\cos\omega_q z^{-1} + r^2 z^{-2}}$	Code distortion (if wide)	Pole r	Narrow notches, shallow depth
Freq.-Domain Excision	$x[n] = \mathcal{F}^{-1}\{W[k]\mathcal{F}\{r[n]\}\}$	Latency, artifacts	Window N, overlap	Adaptive threshold, overlap-add
Beamforming	$\mathbf{w} = \mathbf{R}^{-1}\mathbf{C}(\mathbf{C}^H\mathbf{R}^{-1}\mathbf{C})^{-1}\mathbf{f}$	Null errors	Cov. $\mathbf{R}$	Diagonal load, adapt update

heavy-tailed errors while retaining near-optimal efficiency in Gaussian regimes. (30)

Integrity monitoring quantifies the probability that an undetected failure leads to hazardously misleading information. A protection level can be expressed as

$$\mathrm{PL} = K_\alpha \sqrt{\mathbf{g}^T \left(\mathbf{H}^T\mathbf{R}^{-1}\mathbf{H}\right)^{-1} \mathbf{g}} + \delta_{\mathrm{bias}},$$

where $\mathbf{g}$ maps state covariance to a scalar risk metric of interest (e.g., horizontal position), K_α is a threshold multiplier derived from a continuity risk allocation, and δ_{bias} is a bound on residual biases from multipath and ionosphere after mitigation. Receiver autonomous consistency checks compare redundant subsets of measurements to flag anomalies; a parity space norm

$$\pi = |0\mathbf{S}\mathbf{y}|0, \qquad \mathbf{S} = \mathbf{I} - \mathbf{H}\left(\mathbf{H}^T\mathbf{H}\right)^{-1}\mathbf{H}^T,$$

serves as a change detector whose distribution under nominal conditions sets detection thresholds as a function of false alarm rate.

Tracking loops face the classic trade among noise, dynamics, and nonlinearity. The steady-state code tracking jitter is approximated by

$$\sigma_\tau^2 \approx \frac{B_n}{2C/N_0}\frac{1}{G_c^2 T_I} + \sigma_{\mathrm{mp}}^2 + \sigma_{\mathrm{iono}}^2 + \sigma_{\mathrm{int}}^2,$$

where the first term is thermal noise gated by discriminator slope and integration time, and the remaining terms aggregate multipath, ionospheric, and interference contributions. Carrier tracking exhibits analogous decomposition with different mapping from C/N_0 to phase jitter due to loop order and bandwidth.

8. Experimental Methodology and Benchmarking Protocol

A fair comparison across receiver designs and algorithms rests on a methodology that blends repeatable channel emulation with representative field recordings (31). The channel emulation path constructs baseband signals with controllable geometry, clock dynamics, and propagation effects. Satellite ephemerides drive signal time-of-transmission, while the receiver trajectory supplies time-of-reception, allowing precise truth for ranges and Dopplers. Multipath is injected through a collection of specular paths with geometry derived from simplified urban canyons and through diffuse tails following exponential or measured delay profiles. Ionospheric delays are constructed from parametric thin-shell models perturbed by stochastic processes to emulate storm-time variability, and scintillation is synthesized using phase screens with tunable structure functions. Interference is superposed as tones, combs, and band-limited white processes, with onset times and power ramps chosen to examine both steady-state and transient responses.

Field recordings complement emulation by exposing receivers to unmodeled effects. Rooftop, street-level, and maritime sites are selected to span elevation masks and scattering environments. Antenna configurations include single-patch, choke-ring, and arrays with element patterns measured in anechoic chambers to align spatial processing assumptions with reality (32). Front-ends are set to several noise bandwidths to study the correlation between filtering and multipath bias envelopes, and sampling rates are chosen to control aliasing of interference products that might otherwise be misattributed to correlation distortions.

Metrics focus on observables and navigation outputs that reflect both precision and resilience. For code and carrier tracking, lock probability is measured as the fraction of epochs where discriminator outputs remain within predefined capture bounds, and cycle-slip events are detected with geometry-free phase combinations and time-differenced checks. Pseudorange residual distributions are analyzed for skewness and kurtosis as indicators of multipath-dominated regimes, while carrier residuals are examined for heavy tails indicative of scintillation or coherent interference. Navigation solutions are summarized by root-mean-square position error, time-transfer Allan deviation over averaging factors spanning subsecond to kilosecond, and outage statistics defined as time fractions exceeding error thresholds relevant to specific applications.

To mitigate confounding, experiments vary one impairment class at a time while holding others constant before introducing overlaps. For example, narrowband interference is stepped in jammer-to-signal ratio across a fixed multipath scene to separate spectral notch efficacy from correlation shape distortions. Similarly, scintillation parameters are increased while maintaining constant geometry to isolate loop bandwidth and integration time effects on carrier robustness. Crossvalidation is used where possible: the same impairment sequences are replayed through distinct receiver implementations to assess algorithm rather than codebase idiosyncrasies. (33)

9. Results and Discussion

Under isolated multipath with a dominant specular component at delays shorter than one chip, code biases rise in proportion to reflection amplitude and exhibit oscillatory dependence on relative phase, consistent with the bias approximations derived earlier. Narrower correlator spacings reduce envelope peaks but elevate thermal noise contributions, producing a trade where the minimum mean-squared error occurs at a spacing that depends on environment and C/N_0. In scenes with longer-delay reflections, carrier tracking remains largely unaffected in phase but may experience amplitude fading that, combined with Doppler dynamics, reduces loop effective SNR. Receivers using multipath estimating correlators demonstrate reduced biases in stable reflections but occasionally increase jitter when reflections vary quickly, suggesting that adaptation rates require careful tuning.

Ionosphere-driven group delay variations are largely removed by dual-frequency combinations, though the increased measurement noise manifests as a modest degradation in pseudorange precision. During synthetic scintillation with structure functions representative of strong events, carrier loops in second order with low damping present higher cycle-slip probabilities compared to third order or bandwidth-adaptive implementations. Filters that adapt B_n according to instantaneous C/N_0 and discriminator curvature maintain lock through moderate fades with lower slip rates. The state-space estimation of slant delays aids single-frequency modes by borrowing strength across satellites and time; however, when scintillation introduces rapid phase perturbations beyond the assumed process noise, residuals become heavy-tailed and robust weighting improves stability.

Interference tests using single and multiple tones show that time-domain notch filters recover tracking at tone offsets that coincide with code spectrum notches, but introduce correlation distortions when notches broaden to attack high-power tones (34). Frequency-domain excision with short-window FFTs offers more graceful degradation over a range of tone frequencies, at the expense of computational load. Wideband interference raises post-correlation variance and thus tracking jitter; automatic gain control responses that compress to maintain front-end linearity can inadvertently alter correlation shapes if their time constants are comparable to integration times, advocating for AGC designs with dynamics outside the loop bandwidth range.

Spoofing scenarios with small code and Doppler offsets generate asymmetric correlator signatures that detectors using signed area features identify reliably when integration times exceed several milliseconds. During slow pull-off maneuvers, some delay discriminators exhibit tracking to the counterfeit correlation peak, with divergence from inertially aided predictions serving as a secondary cue. Arrays add strong leverage: eigenvalue-based tests trigger earlier in the presence of directional inconsistencies, and constrained beamforming maintains authentic signal gain while reducing spoofer influence until geometric divergence grows too large.

Across combined impairment scenarios, the interactions exhibit additive and sometimes multiplicative effects. For example, multipath with short delays makes code discriminators more sensitive to narrowband interference, since small frequency biases alter the symmetry of the already distorted correlation function. Similarly, scintillation-induced fades reduce effective SNR and create windows where interference detection becomes less reliable due to increased variance in energy detectors (35). Robust filters that downweight outliers tolerate these overlaps better, albeit with a cost in nominal efficiency. Protection levels reflect these interactions: nominal conditions yield tight bounds based on measurement covariance alone, while combined scenarios require explicit bias allowances that can dominate risk budgets, making conservative integrity assurances depend critically on environmental characterization.

Quantitatively, in moderate urban multipath synthesized from measured delay profiles, code RMS errors increase by a factor proportional to scene reflectivity, observed here as approximately linear in a reflectivity index over the examined range. Employing narrower correlator spacings reduces median bias by noticeable margins while increasing jitter, with a net position error reduction that varies with satellite geometry and C/N_0. Under strong amplitude scintillation, lock outages grow to several time intervals per hundred seconds in second-order loops without bandwidth adaptation, while third-order loops with adaptive bandwidths reduce outages substantially. Narrowband tones at moderate jammer-to-signal ratios generate bias-like terms in prompt correlators; detectors based on quadratic forms achieve low false alarm rates at detection probabilities that rise quickly with tone power, while time-frequency approaches provide earlier warnings during ramps.

The composite perspective that emerges is that no single mitigation dominates across all conditions. Narrow correlator spacings, ionosphere-free combinations, and adaptive interference suppression each contribute benefits with context-dependent costs. The most consistent gains appear when estimation acknowledges bias processes explicitly and when detectors exploit multiple domains of evidence, including symmetry in correlation space, energy concentration in time-frequency space, and spatial structure in array space (36). When those elements work together, receivers traverse a more stable operating envelope across multipath, ionosphere, and interference overlaps, with fewer unanticipated failure modes.

10. Conclusion

The evaluation presented a model-driven and measurement-grounded examination of receiver performance when multipath, ionosphere, and interference occur concurrently. A signal and observation framework clarified how correlator-domain perturbations translate into pseudorange and carrier-phase errors, and how those, in turn, influence navigation states and integrity metrics. Multipath created correlation asymmetries that map into code biases with envelopes governed by front-end bandwidth and correlator spacing; ionospheric dispersion and scintillation added both slowly varying delays and rapid phase fluctuations that challenged tracking loops; interference raised variance, imposed structured biases, or mimicked authentic signals, necessitating detection and mitigation across temporal, spectral, and spatial domains. Estimation approaches that included explicit bias states and robust weighting displayed improved resilience when conditions deviated from Gaussian assumptions, and integrity bounds that incorporated bias allowances offered clearer, if sometimes conservative, assurances. The results underscore trade-offs rather than universal prescriptions, suggesting that design choices should reflect anticipated environments, available computational resources, and required continuity levels. While the scenarios and parameter ranges examined cannot claim exhaustive coverage, the analysis delineates mechanisms by which impairments combine and points to mitigation strategies that maintain stable operation over a wide, but not unlimited, set of conditions. Future work can extend these methods to additional bands, tighter coupling with inertial sensors, and refined environmental models while adhering to the same discipline of transparent assumptions and traceable metrics. (37)

Expanding on these findings reveals that the underlying framework serves not merely as an analytical construct but as a bridge between theory and measurable performance. The signal model defines how baseband representations of satellite transmissions interact with hardware limitations and environmental phenomena. By embedding this within a measurement model that explicitly accounts for bias and noise, the study established a quantitative path from correlator distortions to navigation-level consequences. This linkage is crucial, because traditional performance evaluations often stop at tracking loop metrics—such as signal-to-noise ratio or lock time—without extending to their downstream implications on positioning accuracy and integrity monitoring. The model-driven approach therefore provided a coherent way to understand how specific impairments propagate through the receiver chain, allowing engineers to pinpoint which mechanisms dominate under different operational regimes.

The concurrent occurrence of multipath, ionospheric irregularities, and interference revealed complex interactions that cannot be captured by superposition alone. For instance, when scintillation-induced phase noise interacts with narrowband interference, the resulting error process becomes non-Gaussian, exhibiting heavy tails and temporal clustering. Similarly, multipath reflections can either exacerbate or partially mask interference effects depending on their relative delay and carrier phase. These intertwined behaviors were explored through both controlled simulations and empirical datasets, which highlighted that simple mitigation strategies, when applied in isolation, often fail to generalize (38). The evaluation thus emphasized the need for composite approaches that jointly consider amplitude, phase, and delay domains rather than treating each impairment as separable.

From a signal-processing standpoint, the findings illustrate the value of adaptive estimation frameworks. Traditional tracking architectures assume stationary error statistics, yet real-world impairments violate that assumption. By introducing dynamic bias states and adaptive noise covariance updates, the receiver gains the ability to respond to transient anomalies without losing stability. The Kalman filter variants tested under these conditions demonstrated that including bias states allowed for a more faithful estimation of code and carrier measurements, reducing unmodeled residuals that would otherwise appear as apparent noise. However, this came at the cost of increased computational load and parameter tuning complexity, reaffirming that robustness and efficiency often pull in opposite directions. When operating in constrained hardware platforms, designers must therefore decide how much adaptivity can be afforded before latency or power budgets are compromised.

The investigation into integrity metrics provided additional insight (39). Integrity, in this context, denotes the confidence that navigation solutions remain within acceptable error bounds, even in the presence of unmodeled biases. Conventional approaches assume that residual errors follow Gaussian statistics with known variance; however, the concurrent impairment environment violates that assumption by introducing both systematic biases and nonstationary noise. By augmenting integrity monitoring with explicit bias allowances, the evaluation showed that protection levels could be maintained without over-optimistic assumptions. These expanded bounds were indeed more conservative, but they provided transparency about the level of uncertainty introduced by environmental dynamics. Such conservatism is not a flaw but a design choice aligned with safety-critical applications, where false integrity declarations can have severe consequences.

The measurement-driven component of the evaluation served as both validation and calibration for the analytical models. Field experiments in controlled test ranges, along with data from urban and maritime trials, provided empirical evidence that supported the theoretical predictions. In several instances, observed correlation distortions and phase jitter closely matched those forecasted by the model when realistic front-end parameters were applied. Deviations between model and measurement tended to occur in highly dynamic conditions where unmodeled hardware nonlinearities—such as automatic gain control transients or front-end temperature drift—introduced additional variability (40). These discrepancies underscore the necessity of maintaining traceable metrics that clearly separate model assumptions from empirical observations. Without such transparency, it becomes difficult to distinguish between true environmental effects and instrument artifacts.

Interference mitigation remained one of the most demanding aspects of the study. The evaluation cataloged a range of interference types, from narrowband continuous-wave tones to broadband chirp-like signals, each with distinct temporal and spectral footprints. Time-domain excision methods proved effective for bursty interference, whereas frequency-domain adaptive notching was superior against persistent narrowband sources. Spatial filtering using controlled antenna arrays offered further suppression but required calibration to avoid introducing phase bias into carrier tracking. Importantly, the presence of interference changed not only noise statistics but also the apparent structure of multipath and scintillation effects, suggesting that mitigation algorithms must be context-aware. The measurement campaigns demonstrated that uncoordinated filtering could inadvertently degrade navigation performance by distorting the signal spectrum, reinforcing the need for joint optimization across domains. (41)

Ionospheric effects, though well understood in isolation, revealed subtle coupling with receiver dynamics when examined in concert with other impairments. Scintillation-induced amplitude fading can cause intermittent loss of lock, leading to cycle slips in carrier phase tracking. These slips propagate directly into ambiguities that corrupt high-precision positioning solutions such as real-time kinematic (RTK) or precise point positioning (PPP). The evaluation's model-based component captured this behavior through stochastic differential equations representing phase noise evolution, while the measurement component confirmed the frequency dependence predicted by theory. Mitigation strategies based on adaptive loop bandwidth control were found to be partially effective, stabilizing tracking under moderate scintillation but failing under deep fades. The study concluded that multi-frequency and multi-constellation diversity remains the most practical means to counter severe ionospheric variability, a finding consistent with operational experience.

Multipath remained a dominant source of bias across all tested environments. The model demonstrated that the shape and magnitude of the correlation distortion depend strongly on the front-end bandwidth and correlator spacing configuration. Wideband receivers with narrow correlator spacing exhibited superior multipath discrimination but demanded higher sampling rates and processing power (42). Conversely, reduced-bandwidth designs sacrificed some multipath resilience for improved noise suppression and power efficiency. The evaluation thus reinforced the principle that no single configuration is optimal across all conditions. Receiver designers must tailor their architectures to the anticipated propagation environment, balancing between hardware cost, energy consumption, and accuracy requirements(43).

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